ZIMSEC NOVEMBER 2020

SUGGESTED MARKING GUIDE

PURE MATHEMATICS **6042/1**

PAPER 1

Prepared by Tarakino Nyasha Patrick (Trockers)
0772978155/0717267175
ntarakino@gmail.com

TROCKERS

NB: If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the **nearest degree**, and in other cases it should be given correct to **2 significant figures.**

1) $3x - 4y + 2 = 0$

$\Rightarrow 4y = 3x + 2$

$\Rightarrow y = \frac{3}{4}x + \frac{1}{2}$

$\Rightarrow Gradient = \frac{3}{4}$

Parallel lines have the same gradient

Now:

$Gradient = \frac{3}{4};\ Point = (2;\ -3)$

$\Rightarrow \frac{y+3}{x-2} = \frac{3}{4}$

$\Rightarrow 4(y+3) = 3(x-2)$

$\Rightarrow 4y + 12 = 3x - 6$

$\Rightarrow 3x - 4y - 6 - 12 = 0$

$\therefore 3x - 4y - 18 = 0$

2) $2^{3x-2} = 6$

$\Rightarrow In(2^{3x-2}) = In6$

$\Rightarrow (3x-2)In2 = In6$

$\Rightarrow 3x - 2 = \frac{In6}{In2}$

$\Rightarrow 3x = \frac{In6}{In2} + 2$

$\Rightarrow x = \frac{1}{3}\left(\frac{In6}{In2} + 2\right)$

$\Rightarrow x = 1.528320834$

$\therefore x = 1.53\ (to\ 3.s.f)$

3) $x = cos2t$ $\qquad$ $y = sin2t$

$\Rightarrow \frac{dx}{dt} = -2sin2t$ $\qquad$ $\Rightarrow \frac{dy}{dt} = 2cos2t$

Now:

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

$\Rightarrow \frac{dy}{dx} = -\frac{2cos2t}{2sin2t}$

$\therefore \frac{dy}{dx} = -cot2t$

4) Given

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

Now

$$\sum_{r=10}^{50} r^2 = \sum_{r=1}^{50} r^2 - \sum_{r=1}^{9} r^2$$

$$= \frac{1}{6}(50)(50+1)(50\times 2+1) - \frac{1}{6}(9)(9+1)(2\times 9+1)$$

$$= \frac{1}{6}(50)(51)(101) - \frac{1}{6}(9)(10)(19)$$

$$= 42\ 925 - 285$$

$$= 42\ 640$$

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5) (a) $\frac{a^{-\frac{3}{2}} \times a^{\frac{3}{4}}}{a^{-\frac{3}{4}}}$

$= \frac{a^{\frac{3}{4}-\frac{3}{2}}}{a^{-\frac{3}{4}}}$

$= \frac{a^{-\frac{3}{4}}}{a^{-\frac{3}{4}}}$

$= a^{-\frac{3}{4}+\frac{3}{4}}$

$= a^0$

$= 1$

(b) $\left(\frac{125a^3}{27b^6}\right)^{-\frac{1}{3}}$

$= \frac{1}{\left(\frac{125a^3}{27b^6}\right)^{\frac{1}{3}}}$

$= \sqrt[3]{\frac{27b^6}{125a^3}}$

$= \frac{3b^2}{5a}$

6) $x^2 + y^2 - 2x - 6y + 1 = 0$

$\Rightarrow 2x + 2y\frac{dy}{dx} - 2 - 6\frac{dy}{dx} = 0$

$\Rightarrow x + y\frac{dy}{dx} - 1 - 3\frac{dy}{dx} = 0$

$\Rightarrow (y-3)\frac{dy}{dx} = 1 - x$

$\Rightarrow \frac{dy}{dx} = \frac{1-x}{y-3}$

$\Rightarrow \frac{dy}{dx}_{(x,y)=(1,0)} = \frac{1-1}{0-3}$

$\Rightarrow \frac{dy}{dx}_{(x,y)=(1,0)} = 0$

$\therefore$ The gradient of the circle at the point $(1; 0) = 0$

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7) a) $f(x) = \sqrt{3}sinx + cosx$

Now:

$\sqrt{3}sinx + cosx \equiv Rcos(x - \alpha)$

$\equiv Rcosxcos\alpha + Rsinxsin\alpha$

$\Rightarrow Rcos\alpha = 1$ and $Rsin\alpha = \sqrt{3}$

$\Rightarrow cos\alpha = \frac{1}{R}$ $sin\alpha = \frac{\sqrt{3}}{R}$

So

$cos^2\alpha + sin^2\alpha = 1$

$\Rightarrow \left(\frac{1}{R}\right)^2 + \left(\frac{\sqrt{3}}{R}\right)^2 = 1$

$\Rightarrow \frac{1}{R^2} + \frac{3}{R^2} = 1$

$\Rightarrow \frac{1+3}{R^2} = 1$

$\Rightarrow R^2 = 4$

$\Rightarrow R = \sqrt{4}$

$\therefore R = 2$

Also

$tan\alpha = \frac{sin\alpha}{cos\alpha}$

$\Rightarrow tan\alpha = \frac{\left(\frac{\sqrt{3}}{R}\right)}{\left(\frac{1}{R}\right)}$

$\Rightarrow tan\alpha = \sqrt{3}$

$\Rightarrow \alpha = tan^{-1}(\sqrt{3})$

$\therefore \alpha = \frac{\pi}{3}$

$\therefore f(x) \equiv 2cos\left(x - \frac{\pi}{3}\right)$

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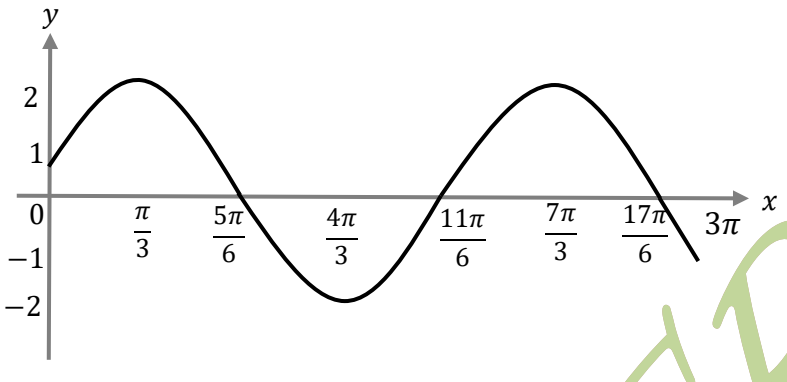
b) $f(x) \equiv 2cos\left(x - \frac{\pi}{3}\right)$ for $0 \leq x \leq 3\pi$

NB: Transformations of the graph of $y = cosx$

- Translation along the $x\text{-}axis$, moving $\frac{\pi}{3}$ units rightwards
- Stretch along the $y\text{-}axis$, multiplying every $y\text{-}coordinate$ by 2.



8) $cos\theta = 2cos2\theta + 1$

$\Rightarrow cos\theta = 2(2cos^2\theta - 1) + 1$

$\Rightarrow cos\theta = 4cos^2\theta - 2 + 1$

$\Rightarrow 4cos^2\theta - cos\theta - 1$

$\Rightarrow cos\theta = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-1)}}{2(4)}$

$\Rightarrow cos\theta = \frac{1 \pm \sqrt{17}}{8}$

$\Rightarrow \theta = cos^{-1}\left(\frac{1 + \sqrt{17}}{8}\right)$

$\Rightarrow \theta = 50.17922707°; \quad 360° - 50.17922707°$

$\Rightarrow \theta = 50.17922707°; \quad 309.8207729°$

$\Rightarrow \theta = 50.2°; \quad 309.8°$

or

$\theta = cos^{-1}\left(\frac{1 - \sqrt{17}}{8}\right)$

$\Rightarrow \theta = 112.9786567°; \quad 360° - 112.9786567°°$

$\Rightarrow \theta = 112.9786567°; \quad 247.0213433°$

$\Rightarrow \theta = 113{,}0°; \quad 247.0°$

$\therefore \theta = 50.2°; \quad 113{,}0°; \quad 247.0°; \quad 309.8$

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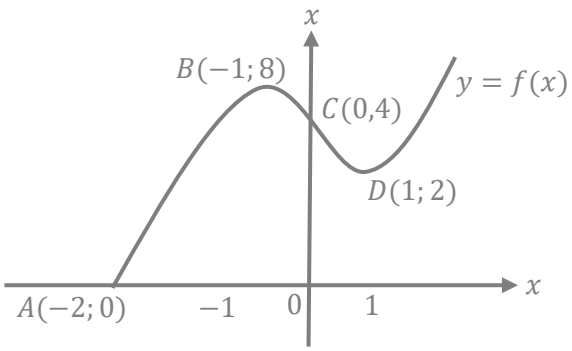
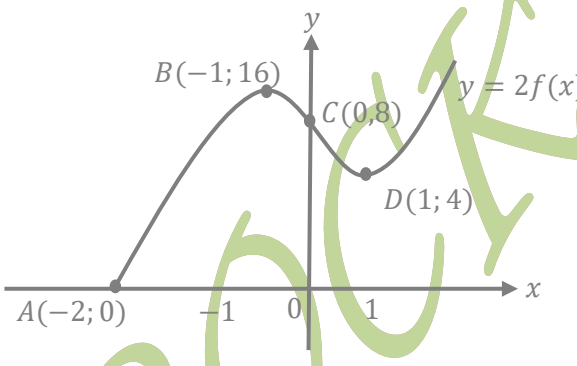
9) Given the graph of the function $f(x)$



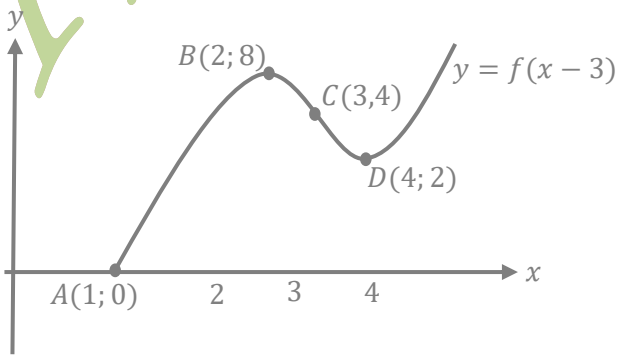
(a) $2f(x)$

- Stretch along the $y\text{-}axis$, multiplying every $y\text{-}coordinate$ by 2.



(b) $f(x-3)$

- Translation along the $x\text{-}axis$, moving 3 units rightwards



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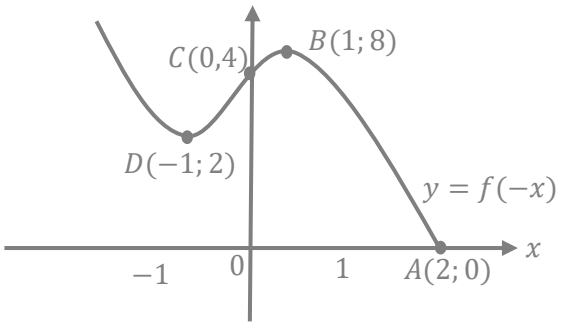
(c) $f(-x)$

- Reflection along the line $x = 0$ or $y\text{-}axis$



10) (a) $Length\ of\ arc = r\theta$

$$= 9\left(\frac{\pi}{6}\right)$$

$$= \frac{3}{2}\pi\ cm$$

(b) $Area\ of\ triangle\ POQ = \frac{1}{2}r^2 sin\theta$

$$= \frac{1}{2}(9)^2 sin\left(\frac{\pi}{6}\right)$$

$$= \frac{81}{2}\left(\frac{1}{4}\right)$$

$$= \frac{81}{4}\ cm^2$$

(c) $Area\ of\ shaded\ segment = \frac{1}{2}r^2(\theta - sin\theta)$

$$= \frac{1}{2}(9)^2\left(\frac{\pi}{6} - sin\frac{\pi}{6}\right)$$

$$= \frac{81}{2}\left(\frac{\pi}{6} - \frac{1}{2}\right)$$

$$= \frac{27}{2}\left(\frac{\pi}{2}\right) - \frac{81}{2}\left(\frac{1}{2}\right)$$

$$= \left(\frac{27}{4}\pi - \frac{81}{4}\right)\ cm^2$$

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11) $f: x \longrightarrow x^2 - 4x,\ x \in \mathbb{R}$

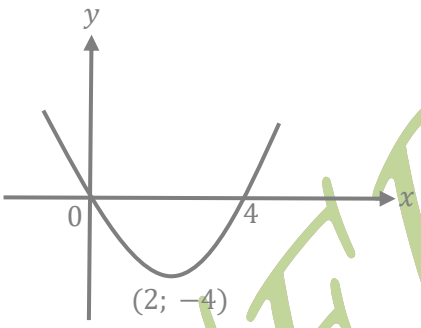
$\Rightarrow f: x \longrightarrow x^2 - 4x + (-2)^2 - (-2)^2$

$\Rightarrow f: x \longrightarrow (x-2)^2 - 4$

$*$ *Turning point* $(2;\ -4)$

$*$ When $x = 0, y = 0$ and when $y = 0, x = 0$ *or* 4

(a) The graph of $f(x) = (x-2)^2 - 4$



(b) $f(x) \in \mathbb{R}: f(x) \geq -4$

(c) (i) $k = 2$ *(NB: The line of symmetry)*

(ii) $f(x) = (x-2)^2 - 4$

Let $x = (y-2)^2 - 4$

$\Rightarrow x + 4 = (y-2)^2$

$\Rightarrow y - 2 = \sqrt{x+4}$

$\Rightarrow y = \sqrt{x+4} + 2$

$\therefore f^{-1}(x) = \sqrt{x+4} + 2, \qquad x \in \mathbb{R}: x \geq -4$

12) $p(x) = 6x^3 - 11x^2 + ax + b$

(a) $p(-1) = -24$

$\Rightarrow 6(-1)^3 - 11(-1)^2 + a(-1) + b = -24$

$\Rightarrow -6 - 11 - a + b = -24$

$\Rightarrow -17 - a + b = -24$

$\Rightarrow -a + b = -24 + 17$

$\Rightarrow -a + b = -7$

$\Rightarrow a - b = 7$ (i)

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Also

$p(1) = 0$

$\Rightarrow 6(1)^3 - 11(1)^2 + a(1) + b = 0$

$\Rightarrow 6 - 11 + a + b = 0$

$\Rightarrow -5 + a + b = 0$

$\Rightarrow a + b = 0 + 5$

$\Rightarrow a + b = 5$ (ii)

Equation (i) + equation (ii):

$\Rightarrow 2a = 12$

$\therefore a = 6$

$a + b = 5$ (ii)

$\Rightarrow 6 + b = 5$

$\therefore b = -1$

(b) $p(x) = 6x^3 - 11x^2 + 6x - 1$

$$
\begin{array}{r|l}
 & 6x^2 - 5x + 1 \\
\hline
x - 1 & 6x^3 - 11x^2 + 6x - 1 \\
 & -(6x^3 - 6x^2) \\
\hline
 & -5x^2 + 6x - 1 \\
 & -(-5x^2 + 5x) \\
\hline
 & x - 1 \\
 & -(x - 1) \\
\hline
 & 0
\end{array}
$$

$\Rightarrow p(x) = (x - 1)(6x^2 - 5x + 1)$

$= (x - 1)(6x^2 - 3x - 2x + 1)$

$= (x - 1)[3x(2x - 1) - 1(2x - 1)]$

$= (x - 1)(3x - 1)(2x - 1)$

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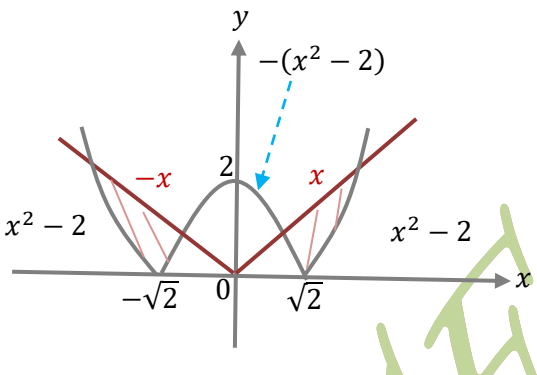
(c) $p(x) = 0$

$\Rightarrow (x-1)(3x-1)(2x-1) = 0$

$\therefore x = \frac{1}{3}; \frac{1}{2} \ or \ 1$

13) (a) Sketching the graphs of $y = |x^2 - 2|$ and $y = |x|$



(b) $|x^2 - 2| = |x|$

Hence means use the graphs

$\Rightarrow -(x^2 - 2) = -x$ or $-(x^2 - 2) = x$

$\Rightarrow -x^2 + 2 = -x$ or $-x^2 + 2 = x$

$\Rightarrow x^2 - x - 2 = 0$ or $x^2 + x - 2 = 0$

$\Rightarrow x^2 - 2x + x - 2 = 0$ or $x^2 + 2x - x - 2 = 0$

$\Rightarrow x(x-2) + 1(x-2) = 0$ or $x(x+2) - 1(x+2) = 0$

$\Rightarrow (x+1)(x-2) = 0$ or $(x-1)(x+2) = 0$

$\therefore x = -1 \ or \ 2$ or $x = -2 or \ 1$

(c) Hence means use the graphs

$x \in \mathbb{R}: -2 < x < -1 \ \cup \ 1 < x < 2 \ (Shaded \ region)$

14) $f: x \longrightarrow x^2 - 2x + 2, \ x \in \mathbb{R}$ and $g: x \longrightarrow x + 3, x \in \mathbb{R}$

(a) $f(x) > 10$

$\Rightarrow x^2 - 2x + 2 > 10$

$\Rightarrow x^2 - 2x - 8 > 0$

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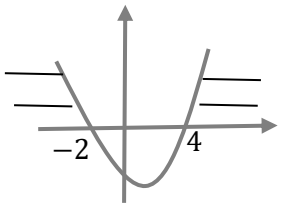
$\Rightarrow x^2 - 4x + 2x - 8 > 0$

$\Rightarrow x(x-4) + 2(x-4) > 0$

$\Rightarrow (x-4)(x+2) > 0$

$\Rightarrow$ The critical values are $x = -2 \; or \; x = 4$



$x \in \mathbb{R}: x < -2 \cup x > 4 \; (Shaded \; region)$

(b) $f: x \longrightarrow x^2 - 2x + 2, \; x \in \mathbb{R}$

$\Rightarrow f: x \longrightarrow x^2 - 2x + (-1)^2 - (-1)^2 + 2$

$\Rightarrow f: x \longrightarrow (x-1)^2 + 1$

$\therefore$ The range is $f(x) \in \mathbb{R}: f(x) \geq 1$

f has no inverse since it is not injective or it is not a one to one function.

(c) $gf(x) = f(x) + 3$

$= x^2 - 2x + 2 + 3$

$= x^2 - 2x + 5$

Now

$gf(x) = 0$

$\Rightarrow x^2 - 2x + 5 = 0$

So

$b^2 - 4ac = (-2)^2 - 4(1)(5)$

$= 4 - 20$

$= -16$

Since $b^2 - 4ac < 0 \; \therefore$ the equation $gf(x) = 0$ has no real roots.

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15) (a) Expanding

$$\frac{1-y}{\sqrt{4-y}}.$$

$$\frac{1-y}{\sqrt{4-y}} = \frac{1-y}{(4-y)^{\frac{1}{2}}} = (1-y)(4-y)^{-\frac{1}{2}}$$

Aside:

$$(4-y)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}\left(1-\frac{y}{4}\right)^{-\frac{1}{2}} = \frac{1}{2}\left(1-\frac{y}{4}\right)^{-\frac{1}{2}}$$

$$(1+n)^x = 1 + nx + \frac{n(n-1)}{2!}x^2$$

$$\frac{1}{2}\left(1-\frac{y}{4}\right)^{-\frac{1}{2}} = \frac{1}{2}\left[1 + \left(-\frac{1}{2}\right)\left(-\frac{y}{4}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2\times 1}\left(-\frac{y}{4}\right)^2\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{8}y + \frac{3}{8}\left(\frac{y^2}{16}\right)\right]$$

$$= \frac{1}{2}\left(1 + \frac{1}{8}y + \frac{3}{128}y^2\right)$$

$$= \frac{1}{2} + \frac{1}{16}y + \frac{3}{256}y^2$$

Now

$$\frac{1-y}{\sqrt{4-y}} = (1-y)\left(\frac{1}{2} + \frac{1}{16}y + \frac{3}{256}y^2\right)$$

$$= \frac{1}{2} + \frac{1}{16}y + \frac{3}{256}y^2 - \frac{1}{2}y - \frac{1}{16}y^2 - \cdots$$

$$= \frac{1}{2} - \frac{7}{16}y - \frac{13}{256}y^2$$

(b) Given $y = \frac{2}{5}$

$$\frac{1-y}{\sqrt{4-y}} = \frac{1-\frac{2}{5}}{\sqrt{4-\frac{2}{5}}}$$

$$= \frac{\frac{3}{5}}{\sqrt{\frac{18}{5}}}$$

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$$= \frac{3}{5} \times \sqrt{\frac{5}{18}}$$

$$= \frac{3}{5} \times \sqrt{\frac{5}{9 \times 2}}$$

$$= \frac{3}{5} \times \frac{1}{3} \times \frac{\sqrt{5}}{\sqrt{2}}$$

$$= \frac{1}{5} \times \frac{\sqrt{5}(\sqrt{2})}{\sqrt{2}(\sqrt{2})}$$

$$= \frac{1}{5} \times \frac{\sqrt{10}}{2}$$

$$= \frac{\sqrt{10}}{10} \ (As\ required)$$

(c) Finding $\sqrt{10}$ given that $y = \frac{2}{5}$

$$\frac{1-y}{\sqrt{4-y}} = \frac{1}{2} - \frac{7}{16}y - \frac{13}{256}y^2$$

$$\Rightarrow \frac{\sqrt{10}}{10} = \frac{1}{2} - \frac{7}{16}\left(\frac{2}{5}\right) - \frac{13}{256}\left(\frac{2}{5}\right)^2$$

$$\Rightarrow \frac{\sqrt{10}}{10} = \frac{507}{1600}$$

$$\Rightarrow \sqrt{10} = 10\left(\frac{507}{1600}\right)$$

$$\therefore \sqrt{10} = \frac{507}{160}$$

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16) (a) $In(5 + e^{-2x}) = 3$

$\Rightarrow e^{In(5+e^{-2x})} = e^3$

$\Rightarrow 5 + e^{-2x} = e^3$

$\Rightarrow e^{-2x} = e^3 - 5$

$\Rightarrow Ine^{-2x} = In(e^3 - 5)$

$\Rightarrow -2x = In(e^3 - 5)$

$\Rightarrow x = -\frac{1}{2}In(e^3 - 5)$

$\Rightarrow x = -1.356868232581$

$\therefore x = -1.36\ (to\ 3\ s.f.)$

b) $x = mn^{-t}$

$\Rightarrow Inx = In(mn^{-t})$

$\Rightarrow Inx = In(m) + In(n^{-t})$

$\Rightarrow Inx = In(m) - tIn(n)$

$\Rightarrow Inx = -[In(n)]t + In(m)$

$\Rightarrow vertical\ axis\ intercept = In(m) \quad and \quad gradient\ is - In(n)$

The gradient $= -2.3$

$\Rightarrow -In(n) = -2.3$

$\Rightarrow n = e^{2.3}$

$\Rightarrow n = 9.974182454815$

$\therefore n = 10.0\ (to\ 2\ s.f.)$

The vertical axis coordinates are $(0; 3)$

$\Rightarrow In(m) = 3$

$\Rightarrow m = e^3$

$\Rightarrow m = 20.085536923188$

$\therefore m = 20\ (to\ 2\ s.f.)$

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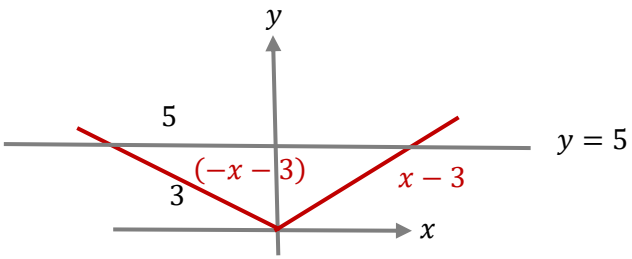
c) $|x-3|<5$



Determining the critical values

$-x+3=5$ $\qquad$ $x-3=5$

$x=-2$ $\qquad$ $x=8$

$\therefore$ The solution set is $x \in \mathbb{R}: -2<x<8$

17) $f(x)=\sqrt{9-x}$

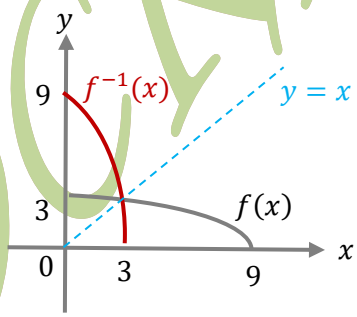
(a) Let $x=\sqrt{9-y}$

$\Rightarrow x^2=9-y$

$\Rightarrow y=9-x^2$

$\therefore f^{-1}(x)=9-x^2, x \in \mathbb{R}: x \geq 0$

(b)



(c) They reflect along the line $y=x$

(d) $\sqrt{9-x}=x$ or $9-9x^2=x$

NB: The graphs intersect at $y=x$

$\Rightarrow 9-x=x^2$

$\Rightarrow x^2+x-9=0$

$\Rightarrow x^2+x=9$

$\Rightarrow x^2+x+\left(\frac{1}{2}\right)^2=9+\left(\frac{1}{2}\right)^2$

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$$\Rightarrow \left(x+\frac{1}{2}\right)^2 = 9+\frac{1}{4}$$

$$\Rightarrow \left(x+\frac{1}{2}\right)^2 = \frac{37}{4}$$

$$\Rightarrow x+\frac{1}{2} = \pm\sqrt{\frac{37}{4}}$$

$$\Rightarrow x = -\frac{1}{2} \pm \sqrt{\frac{37}{4}}$$

NB: The graphs intersect in the first quadrant

$$\therefore x = \frac{-1+\sqrt{37}}{2}$$

18) (a) $f(x) = \frac{3x+4}{(x-4)(x^2-8)}$

$$f(x) = \frac{3x+4}{(x-4)\left(x-\sqrt{8}\right)\left(x+\sqrt{8}\right)}$$

Now:

$$\frac{3x+4}{(x-4)(x^2-8)} \equiv \frac{A}{x-4} + \frac{B}{\left(x-\sqrt{8}\right)} + \frac{C}{\left(x+\sqrt{8}\right)}$$

$$= \frac{A(x^2-8) + B\left(x+\sqrt{8}\right)(x-4) + C\left(x-\sqrt{8}\right)(x-4)}{(x-4)\left(x+\sqrt{8}\right)\left(x-\sqrt{8}\right)}$$

$$= \frac{A(x^2-8) + B\left(x+\sqrt{8}\right)(x-4) + C\left(x-\sqrt{8}\right)(x-4)}{(x-4)(x^2-8)}$$

$$\Rightarrow A(x^2-8) + B\left(x+\sqrt{8}\right)(x-4) + C\left(x-\sqrt{8}\right)(x-4) = 3x+4$$

When $x = 4$: $8A = 16$

$$A = 2$$

When $x = \sqrt{8}$: $2\sqrt{8}\left(\sqrt{8}-4\right)A = 3\sqrt{8}+4$

$$B = \frac{3\sqrt{8}+4}{16-8\sqrt{8}}$$

$$B = \frac{\left(3\sqrt{8}+4\right)\left(16+8\sqrt{8}\right)}{\left(16-8\sqrt{8}\right)\left(16+8\sqrt{8}\right)}$$

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$$B = \frac{(3\sqrt{8}+4)(16+8\sqrt{8})}{(16)^2-(8\sqrt{8})^2}$$

$$B = \frac{48\sqrt{8}+192+64+32\sqrt{8}}{-256}$$

$$B = \frac{256+80\sqrt{8}}{-256}$$

$$B = -\frac{256+160\sqrt{2}}{256}$$

$$B = \frac{-8-5\sqrt{2}}{8}$$

When $x = -\sqrt{8}$: $-2\sqrt{8}(-\sqrt{8}-4)A = -3\sqrt{8}+4$

$2\sqrt{8}(\sqrt{8}+4)A = -3\sqrt{8}+4$

$$C = \frac{-3\sqrt{8}+4}{16+8\sqrt{8}}$$

$$C = \frac{(-3\sqrt{8}+4)(16-8\sqrt{8})}{(16+8\sqrt{8})(16-8\sqrt{8})}$$

$$C = \frac{(-3\sqrt{8}+4)(16-8\sqrt{8})}{(16)^2-(8\sqrt{8})^2}$$

$$C = \frac{-48\sqrt{8}+192+64-32\sqrt{8}}{-256}$$

$$C = \frac{256-80\sqrt{8}}{-256}$$

$$C = -\frac{256-160\sqrt{2}}{256}$$

$$C = \frac{-8+5\sqrt{2}}{8}$$

$$\Rightarrow f(x) = \frac{2}{x-4} + \frac{\frac{-8-5\sqrt{2}}{8}}{(x-\sqrt{8})} + \frac{\frac{-8+5\sqrt{2}}{8}}{(x+\sqrt{8})}$$

$$\therefore f(x) = \frac{2}{x-4} - \frac{8+5\sqrt{2}}{8(x-\sqrt{8})} - \frac{8-5\sqrt{2}}{8(x+\sqrt{8})}$$

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(b) Expanding $f(x)$:

$$f(x) = \frac{2}{x-4} - \frac{8+5\sqrt{2}}{8(x-\sqrt{8})} - \frac{8-5\sqrt{2}}{8(x+\sqrt{8})}$$

$$f(x) = 2(x-4)^{-1} - \frac{8+5\sqrt{2}}{8}(x-\sqrt{8})^{-1} - \frac{8-5\sqrt{2}}{8}(x+\sqrt{8})^{-1}$$

$$f'(x) = -2(x-4)^{-2} + \frac{8+5\sqrt{2}}{8}(x-\sqrt{8})^{-2} + \frac{8-5\sqrt{2}}{8}(x+\sqrt{8})^{-2}$$

$$f''(x) = 4(x-4)^{-3} - \frac{8+5\sqrt{2}}{4}(x-\sqrt{8})^{-3} - \frac{8-5\sqrt{2}}{4}(x+\sqrt{8})^{-3}$$

$$f'''(x) = -12(x-4)^{-4} + \frac{24+15\sqrt{2}}{4}(x-\sqrt{8})^{-4} + \frac{24-15\sqrt{2}}{4}(x+\sqrt{8})^{-4}$$

$$f(0) = 2(0-4)^{-1} - \frac{8+5\sqrt{2}}{8}(0-\sqrt{8})^{-1} - \frac{8-5\sqrt{2}}{8}(0+\sqrt{8})^{-1}$$

$$= -\frac{2}{4} + \frac{8+5\sqrt{2}}{8\sqrt{8}} - \frac{8-5\sqrt{2}}{8\sqrt{8}}$$

$$= -\frac{2}{4} + \frac{5\sqrt{2}}{8\sqrt{8}} + \frac{5\sqrt{2}}{8\sqrt{8}}$$

$$= -\frac{1}{2} + \frac{5\sqrt{2}}{4\sqrt{8}}$$

$$= -\frac{1}{2} + \frac{5\sqrt{2}}{8\sqrt{2}}$$

$$= -\frac{1}{2} + \frac{5}{8}$$

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$$= \frac{1}{8}$$

$$f'(0) = -2(0-4)^{-2} + \frac{8+5\sqrt{2}}{8}\left(0-\sqrt{8}\right)^{-2} + \frac{8-5\sqrt{2}}{8}\left(0+\sqrt{8}\right)^{-2}$$

$$= -\frac{2}{16} + \frac{8+5\sqrt{2}}{64} + \frac{8-5\sqrt{2}}{64}$$

$$= -\frac{2}{16} + \frac{8}{32}$$

$$= \frac{1}{8}$$

$$f''(0) = 4(0-4)^{-3} - \frac{8+5\sqrt{2}}{4}\left(0-\sqrt{8}\right)^{-3} - \frac{8-5\sqrt{2}}{4}\left(0+\sqrt{8}\right)^{-3}$$

$$= -\frac{4}{64} + \frac{8+5\sqrt{2}}{32\sqrt{8}} - \frac{8-5\sqrt{2}}{32\sqrt{8}}$$

$$= -\frac{4}{64} + \frac{5\sqrt{2}}{16\sqrt{8}}$$

$$= -\frac{4}{64} + \frac{5\sqrt{2}}{32\sqrt{2}}$$

$$= -\frac{4}{64} + \frac{5}{32}$$

$$= \frac{3}{32}$$

$$f'''(0) = -12(0-4)^{-4} + \frac{24+15\sqrt{2}}{4}\left(0-\sqrt{8}\right)^{-4} + \frac{24-15\sqrt{2}}{4}\left(0+\sqrt{8}\right)^{-4}$$

$$= -\frac{12}{256} + \frac{24+15\sqrt{2}}{256} + \frac{24-15\sqrt{2}}{256}$$

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$$= -\frac{12}{256} + \frac{24}{128}$$

$$= \frac{9}{64}$$

Now

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

$$f(x) = \frac{1}{8} + \frac{1}{8}x + \frac{\frac{3}{32}}{2 \times 1}x^2 + \frac{\frac{9}{64}}{3 \times 2 \times 1}x^3$$

$$f(x) = \frac{1}{8} + \frac{1}{8}x + \frac{3}{64}x^2 + \frac{3}{128}x^3$$

OR

(a) $f(x) = \frac{3x+4}{(x-4)(x^2-8)}$

$$f(x) = \frac{3x+4}{(x-4)(x^2-8)}$$

Now:

$$\frac{3x+4}{(x-4)(x^2-8)} \equiv \frac{A}{x-4} + \frac{Bx+C}{x^2-8}$$

$$= \frac{A(x^2-8) + (Bx+C)(x^2-8)}{(x-4)(x^2-8)}$$

$$\Rightarrow A(x^2-8) + (Bx+C)(x-4) = 3x+4$$

When $x = 4$: $8A = 16$

$$A = 2$$

Comparing coefficients of:

x^2: $A + B = 0$

$$B = -A$$

x: $-4B + C = 3$

$$8 + C = 3$$

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$$B = -2 \qquad\qquad C = -5$$

$$\Rightarrow f(x) = \frac{2}{x-4} + \frac{-2x-5}{x^2-8}$$

$$\therefore f(x) = \frac{2}{x-4} - \frac{2x+5}{x^2-8}$$

(b) Expanding $f(x)$:

$$f(x) = \frac{2}{x-4} - \frac{2x+5}{x^2-8}$$

$$f(x) = 2(x-4)^{-1} - (2x+5)(x^2-8)^{-1}$$

Aside:

$$(1+n)^x = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \frac{n(n-1)(n-2)(n-4)}{4!}x^4$$

$$(x-4)^{-1} \equiv (-4)^{-1}\left(1-\frac{x}{4}\right)^{-1} \equiv -\frac{1}{4}\left(1-\frac{x}{4}\right)^{-1}$$

Now:

$$-\frac{1}{4}\left(1-\frac{x}{4}\right)^{-1} = -\frac{1}{4}\left[1 + (-1)\left(-\frac{x}{4}\right) + \frac{(-1)(-2)}{2\times 1}\left(-\frac{x}{4}\right)^2 + \frac{(-1)(-2)(-3)}{3\times 2\times 1}\left(-\frac{x}{4}\right)^3\right]$$

$$= -\frac{1}{4}\left[1 + \frac{1}{4}x + \frac{1}{16}x^2 + \frac{1}{64}x^3\right]$$

$$= -\frac{1}{4} - \frac{1}{16}x - \frac{1}{64}x^2 - \frac{1}{256}x^3$$

Also

$$(x^2-8)^{-1} \equiv (-8)^{-1}\left(1-\frac{x^2}{8}\right)^{-1} \equiv -\frac{1}{8}\left(1-\frac{x^2}{8}\right)^{-1}$$

Now:

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$$-\frac{1}{8}\left(1-\frac{x^2}{8}\right)^{-1} = -\frac{1}{8}\left[1+(-1)\left(-\frac{x^2}{8}\right)\right]$$

$$= -\frac{1}{8}\left[1+\frac{1}{8}x^2\right]$$

$$= -\frac{1}{8}-\frac{1}{64}x^2$$

So

$$f(x) = 2(x-4)^{-1} - (2x+5)(x^2-8)^{-1}$$

$$= 2\left(-\frac{1}{4}-\frac{1}{16}x-\frac{1}{64}x^2-\frac{1}{256}x^3+\cdots\right)-(2x+5)\left(-\frac{1}{8}-\frac{1}{64}x^2+\cdots\right)$$

$$= -\frac{1}{2}-\frac{1}{8}x-\frac{1}{32}x^2-\frac{1}{128}x^3+\cdots-\left(-\frac{1}{4}x-\frac{1}{32}x^3-\frac{5}{8}-\frac{5}{64}x^2+\cdots\right)$$

$$= -\frac{1}{2}-\frac{1}{8}x-\frac{1}{32}x^2-\frac{1}{128}x^3+\frac{1}{4}x+\frac{1}{32}x^3+\frac{5}{8}+\frac{5}{64}x^2+\cdots$$

$$= -\frac{1}{2}+\frac{5}{8}-\frac{1}{8}x+\frac{1}{4}x-\frac{1}{32}x^2+\frac{5}{64}x^2-\frac{1}{128}x^3+\frac{1}{32}x^3+\cdots$$

$$= \frac{1}{8}+\frac{1}{8}x+\frac{3}{64}x^2+\frac{3}{128}x^3+\cdots$$

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CONSTRUCTIVE COMMENTS ON THE FORM OF THE PRESENTATION, INCLUDING ANY OMISSIONS OR ERRORS, ARE WELCOME.

BEST WISHES

Nyasha P. Tarakino (Trockers)

+263772978155/+263717267175

ntarakino@gmail.com